Prescribing the Postsingular Dynamics of Meromorphic Functions

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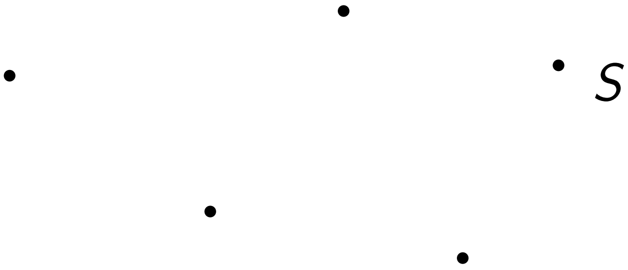

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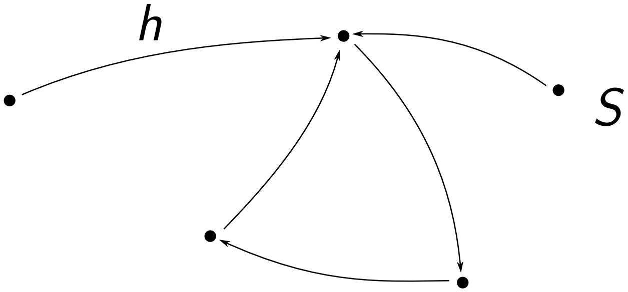

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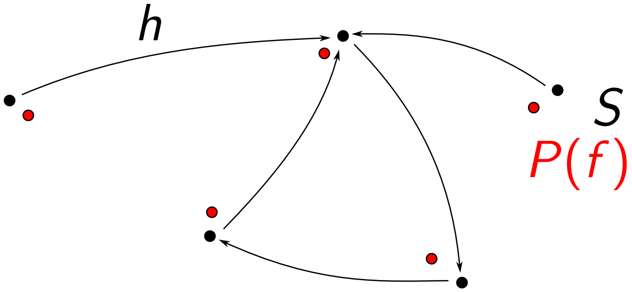

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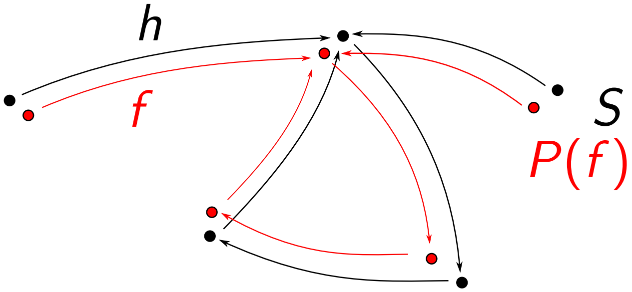

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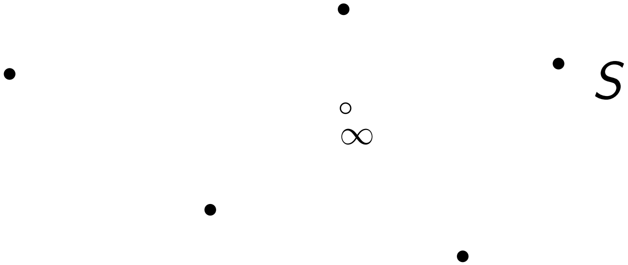

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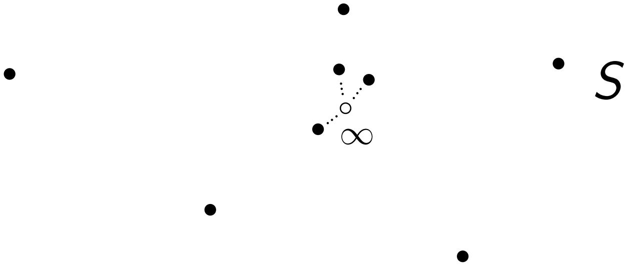

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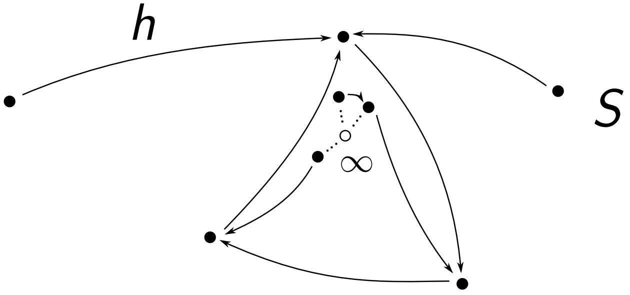

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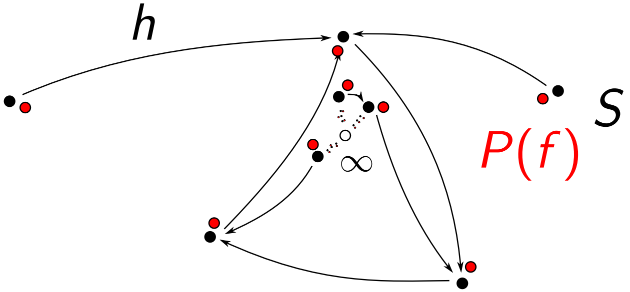

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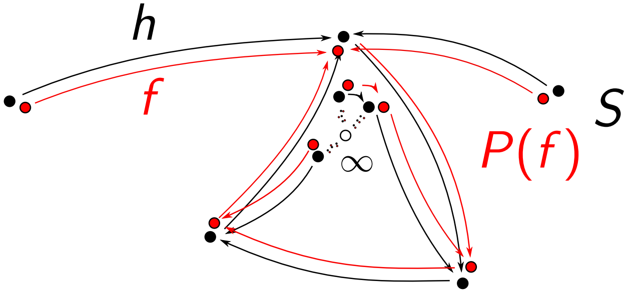

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On the postcritical set of a rational map.
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Question: Given any discrete planar sequence S and some map $h : S \to S$, does there always exist a meromorphic f so that $P(f) = S$, and $f|_S = h$?